

IGNITION OF REACTING SUBSTANCES

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For the solution of problems in the ignition of reacting substances great mathematical difficulties have to be surmounted. An analytic solution is known for only one problem of this type [1]. In the present work with the example of a solution of two different problems in ignition theory, of independent interest, it was shown that all the mathematical difficulties can be overcome by the method of Shvets [2].

§1. We shall consider ignition of a semiinfinite reacting region by a heated medium with other thermal constants under ideal contact conditions. We assume here and below that a zero-order reaction occurs and that all the thermal coefficients are constant. From a solution of this problem, in the special case when $n \rightarrow \infty$, we obtain a solution for the problem examined in [1, 3, 4]. Mathematically the problem reduces to the solution of the following system of equations:

$$\frac{\partial^2 \theta_1}{\partial x^2} = \frac{\partial \theta_1}{\partial \tau} - e^{\theta_1}, \quad x > 0 \quad (1.1)$$

$$\frac{\partial^2 \theta_2}{\partial \xi^2} = \frac{\partial \theta_2}{\partial \tau}, \quad \xi = \left(\frac{\kappa_1}{\kappa_2}\right)^{1/2} x < 0 \quad (1.2)$$

with the following boundary and initial conditions:

$$\begin{aligned} \theta_1|_{x=0} = \theta_2|_{\xi=0}, \quad \frac{\partial \theta_1}{\partial x}|_{x=0} = n \frac{\partial \theta_2}{\partial \xi}|_{\xi=0} \\ \theta_1|_{\tau=0} = \theta_1|_{x=\infty} = -\theta_0, \quad \theta_2|_{\tau=0} = \theta_2|_{\xi=-\infty} = 0 \\ n = \left(\frac{\lambda_2 \rho_2 c_2}{\lambda_1 \rho_1 c_1}\right)^{1/2}, \quad \theta = \frac{(T - T_0) E}{RT_0^2}, \quad x = r \left(\frac{k_0 E}{\lambda_1 RT_0^2} \exp - \frac{E}{RT_0}\right)^{1/2} \\ \tau = \frac{q k_0 E t}{c_1 \rho_1 RT_0^3} \exp - \frac{E}{RT_0}. \end{aligned} \quad (1.3)$$

Here θ is dimensionless temperature, x is a dimensionless coordinate, τ is dimensionless time, T_0 is the initial temperature of the heated nonreacting medium, T° is the initial temperature of the reacting substance, R is the universal gas constant, q is the heat effect of the reaction, r is a dimensionless coordinate, k_0 is a pre-exponential, E is the activation energy, λ is the thermal conductivity coefficient, ρ is the density, c is the heat capacity, t is the time, κ is the thermal diffusivity coefficient, and the indices 1 and 2 refer to the reacting substance and the heated medium respectively.

In the derivation of Eq. (1.1) the Frank-Kamenetskii transformation [5] for $\exp(-E/RT)$ was used. According to [4], Eq. (1.1) satisfactorily describes the ignition of condensed reagents for a first-order reaction if

$$\frac{(T_0 - T^\circ) c_1 \rho_1}{q} \ll 1.$$

To solve the boundary value problem (1.1)-(1.3) we use the method of Shvets [2, 6], which as well as simplicity possesses good convergence. Since in practice a change in temperature occurs in the vicinity of the medium separation boundary, it is appropriate to introduce the thickness of the thermal boundary layer

$\Delta_1(\tau)$ and $\Delta_2(\tau)$. The boundary layers are then at $\pm\infty$, and the conditions at $\tau = 0$ take the following form:

$$\begin{aligned} \theta_1(\Delta_1, \tau) = -\theta_0, \\ \theta_2(-\Delta_2, \tau) = 0, \quad \Delta_1(0) = \Delta_2(0) = 0. \end{aligned} \quad (1.4)$$

As first approximations we shall take the following:

$$\theta_1^{(1)} = a_1 x + b_1, \quad \theta_2^{(1)} = a_2 \xi + b_2. \quad (1.5)$$

We determine the values a_1, a_2, b_1, b_2 from the boundary conditions (1.3) and (1.4):

$$\begin{aligned} a_1 = -\frac{n\theta_0}{\Delta_2 + n\Delta_1}, \\ b_1 = b_2 = -\frac{\theta_0 \Delta_2}{\Delta_2 + n\Delta_1}, \quad a_2 = -\frac{\theta_0}{\Delta_2 + n\Delta_1}. \end{aligned} \quad (1.6)$$

By substituting Eq. (1.5) in the right hand side of Eqs. (1.1) and (1.2) and integrating twice, we obtain:

$$\begin{aligned} \theta_1^{(2)} = 1/6 a_1 x^3 + 1/2 b_1 x^2 - \\ - \exp(b_1 + a_1 x) / a_1^2 + C_1 x + D_1, \end{aligned} \quad (1.7)$$

$$\theta_2^{(2)} = 1/6 a_2 \xi^3 + 1/2 b_2 \xi^2 + C_2 \xi + D_2. \quad (1.8)$$

As before, we obtain the values of C_1, C_2, D_1, D_2 from the initial and boundary conditions (1.3) and (1.4):

$$\begin{aligned} C_2 = (\Delta_2 + n\Delta_1)^{-1} \left[\frac{\gamma}{a_1^3} + \frac{b_1}{2} (\Delta_2^2 - \Delta_1^2) - \right. \\ \left. - \frac{(a_2 \Delta_2^3 + a_1 \Delta_1^3)}{6} - \theta_0 - \frac{1 + a_1 \Delta_1}{a_1^2} e^{b_1} \right], \\ C_1 = \frac{e^{b_1}}{a_1^2} + nC_2, \quad D_2 = C_2 \Delta_2 + \frac{a_2 \Delta_2^3}{6} - \frac{b_1 \Delta_2^2}{2}, \\ D_1 = D_2 + \frac{e^{b_1}}{a_1^2}, \quad \gamma = e^{-\theta_0}. \end{aligned} \quad (1.9)$$

From the Shvets conditions,

$$\frac{\partial \theta_1^{(2)}}{\partial x} \Big|_{x=\Delta_1} = \frac{\partial \theta_2^{(2)}}{\partial \xi} \Big|_{\xi=-\Delta_2} = 0 \quad (1.10)$$

we obtain two ordinary nonlinear differential equations for Δ_1 and Δ_2 :

$$\begin{aligned} a_1 \Delta_1^2 \left(\frac{\Delta_2}{2} + \frac{n\Delta_1}{3}\right) + b_1 (\Delta_1 \Delta_2 + \\ + \frac{n\Delta_1^2}{2} + \frac{n\Delta_2^2}{2}) - \frac{n\Delta_2^3 a_2}{6} - n\theta_0 + \\ + \frac{(a_1 \Delta_2 - n)}{a_1^3} e^{b_1} + \frac{\gamma [n - a_1 (\Delta_2 + n\Delta_1)]}{a_1^2} = 0, \end{aligned} \quad (1.11)$$

$$\begin{aligned} a_2 \Delta_2^2 \left(\frac{\Delta_2}{2} + \frac{n\Delta_1}{2}\right) - b_1 (n\Delta_1 \Delta_2 + \frac{\Delta_1^2}{2} + \frac{\Delta_2^2}{2}) - \\ - \frac{a_1 \Delta_1^3}{6} - \theta_0 + \frac{\gamma - 1 - a_1 \Delta_1}{a_1^2} e^{b_1} = 0. \end{aligned} \quad (1.12)$$

If the nonreacting medium possesses a very high heat capacity the temperature at the medium boundary remains constant, and $n \rightarrow \infty$. In this case instead of the systems (1.11) and (1.12) we have only one equation, which is integrated in closed form:

$$\Delta_1 = \theta_0 \left\{ \frac{\theta_0}{1-\gamma-\gamma\theta_0} \left[\exp \frac{6(1-\gamma-\gamma\theta_0)\tau}{\theta_0^3} - 1 \right] \right\}^{1/2} \approx \sqrt{6\tau} \left[1 + \frac{3(1-\gamma-\gamma\theta_0)}{2\theta_0^3} \right]. \quad (1.13)$$

In the general case system (1.11) and (1.12) is solved by expansion into the series:

$$\begin{aligned} \Delta_1 &= \alpha_1 \tau^{1/2} + \beta_1 \tau + \delta_1 \tau^{3/2} + \dots, \\ \Delta_2 &= \alpha_2 \tau^{1/2} + \beta_2 \tau + \delta_2 \tau^{3/2} + \dots \end{aligned} \quad (1.14)$$

By substituting Eq. (1.14) in Eqs. (1.11) and (1.12) and equating terms with the same power in τ to zero, for the determination of α_1 and α_2 we obtain a set of two nonlinear algebraic equations, and for the determination of β_1 and β_2 , δ_1 and δ_2 we obtain a set of linear equations. Solution of these equations gives $\alpha_1 = \alpha_2 = \sqrt{6}$, $\beta_1 = \beta_2 = 0$ and, finally,

$$\delta_1 = \frac{(1+n)[(9+9n+4\theta_0)(v-\gamma)-9n\theta_0\gamma]}{\sqrt{6}n^2\theta_0^3} \quad (1.15)$$

$$\left(v = \exp \frac{-\theta_0}{1+n} \right)$$

$$\delta_2 = \frac{(1+n)[9(1+n)(v-\gamma)-n\theta_0(6v+5\gamma)]}{\sqrt{6}n^2\theta_0^3}. \quad (1.16)$$

When $n \rightarrow \infty$ and the values found for α_1 , β_1 , and δ_1 are taken into account, the value of Δ_1 coincides accurately with Eq. (1.13) up to terms containing $\tau^{1/2}$ ($i > 3/2$). The first terms in Eq. (1.14) characterize the growth of the boundary layer for the nonreacting medium. It follows from Eqs. (1.13), (1.15), and (1.16), that the heat of reaction at $\theta_0 \gg 1$ has little effect on the thickness of the boundary layer for average values of τ . If in Eqs. (1.7), (1.8), and (1.14) we put the terms which are dependent on heat of reaction equal to zero and equate $\theta_1(0, \tau)$ and $\partial\theta_1/\partial x$ at $x=0$ to the known accurate values [7], we obtain $\theta_1(0, \tau) = -\theta_0/(1+n)$ for both the accurate and the approximate solutions, and the temperature gradient of the approximate solution at $x=0$ in absolute value exceeds the true value by 1.08 [7]. The warming up time for the reacting system is found from the Zel'dovich condition $\partial\theta_1/\partial x = 0$ at $x=0$ [8]. For $\tau = \tau_*$ we have the following equation,

$$\begin{aligned} \frac{\Delta_1^2 n \theta_0}{2} (\Delta_2 + n \Delta_1) + n \theta_0 \Delta_1 (\Delta_1 \Delta_2 - \Delta_1 \Delta_2) = \\ = \frac{(\Delta_2 + n \Delta_1)^2}{n \theta} (e^{b_1} - \gamma). \end{aligned} \quad (1.17)$$

Substitution of the $\Delta_1(\tau)$ and $\Delta_2(\tau)$ values in Eq. (1.17) and solution of the equation obtained with respect to τ give the warming-up time. An approximate solution of Eq. (1.17) is provided by the following expression,

$$\tau_*^{\circ} = \frac{n^2 \theta_0^2}{4(1+n)^2(v-\gamma)} \quad (1.18)$$

which is obtained from Eq. (1.17), if δ_1 and δ_2 are neglected in comparison with α_1 and α_2 . It is easily seen from Eq. (1.18) that $\tau_*^{\circ} \rightarrow 1/4\theta_0^2$, when $n \rightarrow \infty$, and $\tau_*^{\circ} \rightarrow 0$ when $n \rightarrow 0$, i.e., τ_*° is a nonmonotonic function of n which reaches a maximum at n_* when $n = n_*$. The greater the value of θ_0 , the sharper and higher is the τ_*° maximum and the closer is n_* to zero. When $\theta_0 \gg 1$

we have $n_* \approx 1/(\theta_0 - 2)$. Within the $\theta_0 \rightarrow \infty$, $n_* \rightarrow 0$ limits, $\tau_*^{\circ} \rightarrow \infty$. The nonmonotonicity of τ_*° as a function of n is evidently explained by a specific feature of the Arrhenius function, in that the heat separated from the reaction is not converted to zero even at sufficiently low temperatures. This deficiency of the Arrhenius function was noticed in [9], and the method of intersection was used for its elimination. We eliminate it by the method of Spalding [10] and Rosen [11], by assuming that

$$\exp - \frac{E}{RT} \approx A \left(\frac{T - T^{\circ}}{T_0 - T^{\circ}} \right)^k \quad (A = \text{const}, k = \text{const}). \quad (1.19)$$

In a similar way we find, for the heat liberated, in the form of Eq. (1.19),

$$\tau_{1*} = \frac{k+1}{2} \left(1 + \frac{1}{n} \right)^{k-1} \quad \left(\tau_1 = \frac{k_0 A t}{c_{1p} (T_0 - T^{\circ})} \right). \quad (1.20)$$

In this case as n increases the warming-up time decreases monotonically from ∞ at $n=0$ to $\tau_{1*}^{\circ} = (1/2)(k+1)$ at $n \rightarrow \infty$.

For the limiting case $n \rightarrow \infty$ it is possible to find, within the framework of the Shvets method [2], an accurate value for the warming-up time:

$$\begin{aligned} \tau_* &= \frac{\theta_0^3}{6(1-\gamma-\gamma\theta_0)} \ln \frac{2\theta_0(1-\gamma)}{(2+\gamma)\theta_0 - 3(1-\gamma)} \approx \\ &\approx \frac{\theta_0^3}{4} + \frac{3\theta_0}{16} + \frac{3}{32}. \end{aligned} \quad (1.21)$$

Comparison of Eq. (1.21) with the expression for τ_* as $n \rightarrow \infty$, obtained from Eq. (1.17) by considering δ_1 , showed that they are practically identical. By comparing Eq. (1.21) with the expression for τ_* , which was found [4] from Eq. (1.21) with an electronic computer, we see that its accuracy is completely satisfactory. Thus, for $\theta_0 = 5, 10, 15, 20, 25, 30$ from Eq. (1.21) we have $\tau_* = 7.3, 27, 59, 104, 161, 231$.

To evaluate the accuracy of Eq. (1.18) at average values of n we find τ_* for $n=1$ from Eq. (1.17) by the method of trial and linear interpolation. In this way it was found that

$$\begin{aligned} \tau_* &= 28.8, \quad \theta_1^{(2)}(0, \tau_*) = -1.7, \quad \Delta_1(\tau_*) = 15.8, \quad \Delta_2(\tau_*) = 11.9 \quad (\theta_0 = 5) \\ \tau_* &= 957, \quad \theta_1^{(2)}(0, \tau_*) = -2.6, \quad \Delta_1(\tau_*) = 75.8, \quad \Delta_2(\tau_*) = 68.9 \quad (\theta_0 = 10) \\ \tau_* &= 24.7 e^{10} \quad (\theta_0 = 20), \end{aligned}$$

while from Eq. (1.18) we have respectively $\tau_*^{\circ} = 20.7, 934, 25 e^{10}$. The accuracy of formula (1.18) is therefore completely satisfactory within the framework of our approximations. Knowing the value of τ_* we can easily find with Eq. (1.14) the thickness of the heated layer:

$$\begin{aligned} \Delta_{1*} &= \Delta_1(\tau_*) = \frac{n\theta_0}{2(1+n)} \left(\frac{6}{v-\gamma} \right)^{1/2} \times \\ &\times \left[1 + \frac{(9+9n+4\theta_0)(v-\gamma)-9n\gamma\theta_0}{24(1+n)(v-\gamma)\theta_0} \right] \end{aligned} \quad (1.22)$$

and $\Delta_2(\tau_*)$ while we find $\theta_1(0, \tau_*) = b_2$ from Eq. (1.10). When $n \rightarrow 0$ instead of Eq. (1.22) we must use a formula which can be obtained in a similar way by considering that the heat evolved from the reaction is determined by Eq. (1.19), and the warming-up time by Eq. (1.20). As seen from the calculations given the value $\theta_1^{(2)}(0, \tau)$ increases with increase in τ , and the moment $\tau = \tau_*$ corresponds to a temperature close to zero.

§2. Ignition of reagents was first studied by Zel'dovich [12], who took account of the phase transformations. In [12] he gives an outline of the ignition

process and an approximate calculation method. The ideas in [12] are consolidated and further developed in [13]. Here ignition of an explosive material by a heated plate is considered and account is taken of the heat evolved from the reaction. A solid explosive at initial temperature T^0 was first brought into contact with the heated plate, the temperature of which was $T_0 > T_* > T^0$, where T_* is the phase transformation temperature. Following [12] we suppose that the heat of reaction is liberated in the liquid phase, and that the decomposition reaction is endothermic. We also assume that the solid and liquid phases have the same densities. Mathematically the problem reduces to the solution of Eqs. (1.1) and (1.2) for $x < x_*$ and $x > x_*$ respectively with the following boundary and initial conditions:

$$\begin{aligned} \theta_1(0, \tau) = 0, \quad \left(\beta \frac{\partial \theta_2}{\partial x} - \frac{\partial \theta_1}{\partial x} \right) \Big|_{x=x_*} &= p \frac{dx_*}{d\tau}, \\ \theta_1(x_*, \tau) = \theta_2(x_*, \tau) &= -\theta_*, \\ \theta_2|_{x=\infty} = \theta_2|_{\tau=0} &= -\theta_0, \\ \alpha = \frac{\gamma_1}{\gamma_2}, \quad \beta = \frac{\lambda_2}{\lambda_1}, \quad p = \frac{LE}{c_1 RT_0^2}, \\ \theta_* = \frac{(T_0 - T_*)E}{RT_0^2}, \quad \theta_0 = \frac{(T_0 - T^0)E}{RT_0^2}. \end{aligned} \quad (2.1)$$

Here x_* is a dimensionless coordinate of the transformation front, p is the dimensionless heat of the phase transformation, L is the heat of the phase transformation, and the indices 1 and 2 refer to the liquid and solid phases respectively.

As before we introduce the thickness of the boundary layer, $\Delta(\tau)$. The last of the boundary conditions (2.1) and the initial conditions then take the form

$$\theta_2(\Delta, \tau) = -\theta_0, \quad \Delta(0) = 0, \quad x_*(0) = 0. \quad (2.2)$$

For the initial approximations we take the following linear functions:

$$\theta_1^{(1)} = -\frac{\theta_* x}{x_*}, \quad \theta_2^{(1)} = \frac{(\theta_0 - \theta_*)x}{x_* - \Delta} - \theta_0 - \frac{\Delta(\theta_0 - \theta_*)}{x_* - \Delta}. \quad (2.3)$$

These functions satisfy all the boundary conditions apart from the second of the conditions (2.1). Substituting $\theta_1^{(1)}$ and $\theta_2^{(1)}$ in the right hand side of Eqs. (1.1) and (1.2) and integrating the results with respect to x , we obtain the second approximations:

$$\theta_1^{(2)} = \frac{\theta_* x_*^2 x^3}{6x_*^2} - \frac{x_*^2}{\theta_*} \exp - \frac{\theta_* x}{x_*} + g_1 x + h_1, \quad (2.4)$$

$$\begin{aligned} \theta_2^{(2)} &= \frac{\alpha(\theta_0 - \theta_*)x^2}{2(x_* - \Delta)^2} \times \\ &\times \left[\frac{(\Delta - x_*)x_*}{3} + x_* \Delta - x_* \Delta \right] + g_2 x + h_2. \end{aligned} \quad (2.5)$$

We determine the values of g_1 , g_2 , h_1 , and h_2 , from the boundary conditions (2.1) and (2.3):

$$g_1 = \frac{1}{x_*} \left(\frac{\gamma_1 x_*^2}{\theta_*^2} - \theta_* - \frac{x_*^2}{\theta_*} - \frac{\theta_* x_* x_*}{6} \right), \quad h_1 = 0 \quad (2.6)$$

$$\begin{aligned} g_2 &= \frac{1}{\beta x_*} \left\{ \frac{\alpha 3x_*^2 (\theta_0 - \theta_*)}{(x_* - \Delta)^2} \left[\frac{x_* (x_* + \Delta) - \Delta x_*}{2} \right] + \right. \\ &\left. + x_* x_* \left(p + \frac{\theta_*}{3} \right) - \frac{x_*^2 (1 - \gamma_1 - \gamma_1 \theta_*)}{\theta_*^2} \right\}, \end{aligned} \quad (2.7)$$

$$\begin{aligned} h_2 &= \frac{(\theta_0 - \theta_*)x_*^2}{2(x_* - \Delta)^2} \left[\frac{x_*}{3} (2\Delta + x_*) - \Delta x_* \right] - \\ &- \theta_* - g_2 x_* \quad (\gamma_1 = e^{-\theta_*}). \end{aligned} \quad (2.8)$$

For the determination of Δ and x_* we use the boundary condition (2.2) and the Shvets condition $\partial \theta_2 / \partial x = 0$ at $x = \Delta$. The result for Δ and x_* is a set of two ordinary first-order differential equations:

$$\alpha(x_* - \Delta)(x_* + 2\Delta) + 6 = 0 \quad (2.9)$$

$$\begin{aligned} \frac{\alpha \beta}{2} x_* (\theta_0 - \theta_*) (\Delta + x_*) = \theta_* + \\ + \frac{x_*^2}{\theta_*^2} (1 - \gamma_1 - \gamma_1 \theta_*) - x_* x_* \left(p + \frac{\theta_*}{3} \right). \end{aligned} \quad (2.10)$$

We solve Eqs. (2.9) and (2.10) with the initial conditions (2.2) by expansion into a series, assuming that

$$\begin{aligned} \Delta &= \nu_1 \tau^{1/2} + \nu_2 \tau + \nu_3 \tau^{3/2} + \dots, \\ x_* &= \mu_1 \tau^{1/2} + \mu_2 \tau + \mu_3 \tau^{3/2} + \dots \end{aligned} \quad (2.11)$$

Substituting Eq. (2.11) in Eqs. (2.9) and (2.10) we obtain for the determination of ν_1 and μ_1 a set of two second-order algebraic equations and for the determination of ν_2 , μ_2 , ν_3 , μ_3 , ... a set of linear equations. As a result we find

$$\begin{aligned} \mu_1 &= \left[\frac{12}{\alpha(2m+1)(m-1)} \right]^{1/2}, \quad \nu_1 = m\mu_1, \quad \nu_2 = \mu_2 = 0, \quad (2.12) \\ m &= \frac{1}{4} \left\{ 1 + \frac{3\beta(\theta_0 - \theta_*)}{\theta_*} + \left[9 + \frac{3\beta}{\theta_*} (\theta_0 - \theta_*) + \frac{9\beta^2}{\theta_*^2} (\theta_0 - \theta_*)^2 + \right. \right. \\ &\quad \left. \left. + \frac{48}{\theta_*} \left(p + \frac{\theta_*}{3} \right) \right]^{1/2} \right\}, \\ \mu_3 &= \frac{12(1 - \gamma_1 - \gamma_1 \theta_*) \mu_1}{\theta_*^2 [3\alpha\beta(\theta_0 - \theta_*) (3d + m + 4) + 8(3p + \theta_*)]}, \\ \nu_3 &= d\mu_3, \quad d = \frac{4-m}{8m-5}. \end{aligned} \quad (2.13)$$

The first terms in Eq. (2.11) give a solution of the Stefan problem on the melting of nonreacting media. We have a nonreacting medium when $E \rightarrow \infty$, $\theta_0 \rightarrow \infty$, $\theta_* \rightarrow \infty$. Normally $E \gg 1$, $\theta_0 \gg 1$, $\theta_* \gg 1$. Since ν_3 and $\mu_3 \sim \theta_*^3$ they are small in comparison with ν_1 and μ_1 , and consequently the heat liberated from the reaction does not play a substantial role in the values of Δ and x_* with moderate values for τ . An exact solution of the Stefan problem on melting for a nonreacting medium is given in [7]. For $\alpha = 0.4848$, $\beta = 2.5$, $p = 20.125$, $\theta_0 = 12.25$, $\theta_* = 9.22$ we obtain from [7] $x_* = 0.74 \sqrt{\tau}$, and according to the approximate formulas (2.11), (2.12), and (2.13) we have $x_* \approx 0.72 \sqrt{\tau}$. In this way,

$$\frac{\partial \theta_1}{\partial x} \Big|_{x=0} = -\frac{13.03}{\sqrt{\tau}}, \quad \frac{\partial \theta_1}{\partial x} \Big|_{x=0} \approx -\frac{13.92}{\sqrt{\tau}}. \quad (2.14)$$

Consequently the accuracy of the approximate formulas is completely satisfactory. We notice that the thermal coefficients correspond to a hypothetical explosive with the characteristics of nitroglycol [13], but the densities of both phases were taken to be the same. By fulfilling the Zel'dovich [8] condition with Eq. (2.4), we obtain an equation for the determination of the warming-up time, which is accurate up to terms containing $\tau^i (i > 3/2)$:

$$\frac{(\theta_* - 1 + \gamma_1) \tau}{\theta_*^3} (\mu_1 + \mu_3 \tau^{1/2})^2 = 1 + \frac{\mu_1^2}{12} + \frac{\mu_1 \mu_3 \tau}{3} + \frac{\mu_3^2 \tau^2}{4}. \quad (2.15)$$

By neglecting terms containing μ_3 and μ_3^2 , we obtain an approximate solution to Eq. (2.15),

$$\tau_*^0 = \frac{(12 + \mu_1^2) \theta_*^3}{12(\theta_* - 1 + \gamma_1) \mu_1^2}, \quad (2.16)$$

which can be refined if desired by resorting to the method of the small parameter and taking μ_3 as small. When $p \rightarrow 0$, $\alpha \rightarrow \infty$, $\beta \rightarrow 0$ we obtain the principal term of Eq. (1.21) from Eq. (2.16). Here x_* plays the role of Δ , θ_* plays the role of θ_0 , and the second of the conditions (2.1) is converted into the Shvets condition [2]. With a knowledge of the warming-up time it is possible to calculate any other ignition characteristics. In particular, the amount of heat transferred from the heated plate during the warming-up time, accurate up to terms containing μ_3 , is

$$Q = \frac{(\mu_1^2 + 12) \theta_*^2}{9\mu_1^2} \left[\frac{(12 + \mu_1^2) \theta_*^3}{12(\theta_* - 1)} \right]^{1/2}. \quad (2.17)$$

Comparison of Eqs. (2.16) and (1.21) showed that the phase transformation considerably increases the warming-up time. For the hypothetical explosive introduced above, in particular, τ_* is equal to 180.2 according to Eq. (2.16) and $\tau_* = 21.2$ according to Eq. (1.21), if one supposes in Eq. (1.21) that $\theta_0 = \theta_*$ = 9.22 or $\tau_* = 37.5$ if $\theta_0 = 12.25$. The solution of the problem of explosive ignition by the method of Shvets [2] does not present any particular difficulty if the densities of the phases are very different, but in view of the fact that in this case there is mechanical motion of the liquid phase the calculations become more cumbersome.

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